THESEN.

1. Ein Fehler von $^{1}/_{100}$ des wahren Werthes bildet die Grenze für die wünschenswerthe Genauigkeit, ein Fehler von $^{1}/_{1000}$ des wahren Werthes die Grenze für die mögliche Genauigkeit in der Bestimmung einer physikalischen Constanten; genauer als bis auf $^{1}/_{10000}$ ihres Werthes lässt sich kaum eine physikalische Constante auch nur definiren.

2. Obgleich es verfehlt sein würde, im Verlaufe einer Untersuchung eine vorgefasste Meinung beständig festzuhalten, so ist doch im Beginn der Untersuchung eine solche vorgefasste Meinung nicht nur nicht schädlich, sondern sogar nothwendig.

3. Die untergeordnete Stellung, welche in dem Gymnasialunterricht die Studien mathematischen und naturwissenschaftlichen Inhalts gegenüber den humanistischen Studien einnehmen, ist gerechtfertigt.

Figure 14.1:
As was the custom, Hertz's doctoral examination involved the defense of three theses. In the third, Hertz is prepared to defend that high school instruction should favor the humanities over the sciences.

The Mechanics of Representation and the Problem of Life: The Philosophy of Heinrich Hertz

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Abstract: Darstellungsmechanik und Lebensproblem – Heinrich Hertz als Philosoph

Der Philosoph Heinrich Hertz ist bislang kaum gewürdigt worden. Sein Theorienvergleich im Vorwort zu den *Prinzipien der Mechanik* wird etwa als Hinweis auf die Unterdeterminiertheit von wissenschaftlichen Entscheidungen gewürdigt, in die dann Fragen der begrifflichen Sparsamkeit und Angemessenheit hineinspielen. Ansonsten wird seine philosophische Bedeutung vor allem an seinem Einfluss auf Hermann Cohen, Ernst Cassirer und insbesondere Ludwig Wittgenstein gemessen. – Hier wird nun der Versuch unternommen, die *Prinzipien der Mechanik* als physikalisch-philosophisches Buch zu lesen, in dem die Grenzen der Wissenschaft systematisch bestimmt werden. Die Rekonstruktion des philosophischen Programms beginnt mit der Frage, warum die Eingeschränktheit seines Grundgesetzes der Mechanik von Hertz nicht als Mangel sondern als besondere Tugend seiner Physik dargestellt wird. Die Antwort lautet, dass seine Formulierung des Grundgesetzes die Trennung zwischen belebter und unbelebter Natur wahrt und die voraussagende Wissenschaft auf das Studium der unbelebten Natur beschränkt: Die Mechanik und alle Wissenschaft kann nur beharrliche Systeme beschreiben, die keine Sprünge machen und nicht aus sich heraus spontane Bewegungsimpulse produzieren – somit also keine lebendigen Wesen. Der Grund hierfür findet sich in der Mechanik der Repräsentation oder theoretischen Weltbeschreibung: Die Übereinstimmung von Geist und Natur setzt voraus, dass der menschliche Geist dynamische Modelle entwickelt, die sich weiterdenken und auf Voraussagen hin durchrechnen lassen – die also wie die unterstellten mechanischen Systeme der unbelebten Natur vor allem auf ihrer berechenbaren Bahn verharren.

14.1 A Philosophical Program

A suggestive change took place in a 2008 addition to the *Dictionary of Scientific Biography*. The entry on Heinrich Hertz introduced him as a physicist who lived from 1857 to 1894 until supplementary volume 21 of the *Complete Dictionary of Scientific Biography* began listing his fields as "physics, philosophy". Indeed, one can say that Hertz became recognized as a philosopher in his own right only after the centenary of his death which saw the first scholarly monograph on his work (Buchwald 1994) and a conference on "Heinrich Hertz: Classical Physicist, Modern Philosopher" (Baird, Hughes, Nordmann 1998). Just a few years later, Albrecht Fölsing published Hertz's lectures on *The Constitution of Matter* (Hertz 1999) which underscored his tenacious interest in the clarification and conceptual analysis of fundamental terms like "matter" and "force". In light of all this and more, the philosophical aspects of the *Principles of Mechanics* can no longer be ascribed to the illness that prevented Hertz from engaging in more experimental pursuits. Instead, they now appear to be fully integrated with the physical aspects of that work and as the summation of a life-long pursuit that can be traced back at least to Hertz's earliest student days when he heard Fritz Schultze lecture in Dresden about Kant's epistemology and the notion that the mind creates "*Scheinbilder*" of the world – an idiosyncratic term that would reappear in the Introduction to the *Principles of Mechanics*. Not even two years later, in January 1878, Hertz expressed in a letter to his parents that he was pondering conceptual issues, "and particularly the principles of mechanics (as the very words: force, time, space, motion indicate) can occupy one severely enough" (1977, 77). These conceptual issues obviously stayed with him until the very end of this life. He explicitly recognized their philosophical interest when he encouraged his publisher, in a letter dated November 23, 1893, to include among the potential readers of *The Principles of Mechanics* "the circle of philosophical readers" (quoted in Fölsing 1997, 509).

Until the 1990s, then, Hertz's philosophical significance consisted entirely in his reception by philosophers like Hermann Cohen, Ernst Cassirer and, in particular, Ludwig Wittgenstein. The following analysis avoids such detours and appreciates the *Principles of Mechanics* as a mature text that self-consciously pursues a subtle and comprehensive philosophical as well as physical program that considers the constitutive conditions and limits of science while situating the problem of life beyond these limits.[1] The strategy for showing this is simple enough: Upon a brief reconstruction of Hertz's physical argument in the *Principles of Mechanics* follows an analysis of its underlying philosophical commitments. After developing the fundamental law of mechanics, Hertz himself draws attention to its limitations. In these limitations he sees a specific virtue of his mechanics that goes far beyond the elimination of "force" as a basic concept and thus beyond the virtue merely of parsimony. An investigation of Hertz's reasons for delimiting mechanics and all of science will finally lead us to the insight that an appoprIate representation of mechanics draws upon an appropriate conception of the mechanics of representation.

[1] By focusing on the *Principles of Mechanics*, I am excluding for now other aspects of Hertz's philosophy that are formulated in the context of his work on electric waves and the constitution of matter.

14.2 The Fundamental Law

The full title of Hertz's book is *Die Prinzipien der Mechanik in neuem Zusammenhange dargestellt*. It thus promises to develop a new context for the representation of the principles of mechanics. This new context is geometry and Jesper Lützen has shown with great care that the geometrization of mechanics is the most original achievement of Hertz since the call for a forceless mechanics did by no means originate with him (Lützen 2005, see Nordmann 2007). Another, considerably less formal way to describe this construction of a new context for the representation of the principles of mechanics is to say that Hertz sought out a plan or a system of rules that determines what mechanical motion is, how it can be described, and what a mechanical explanation looks like. Accordingly, he arrives at his single fundamental law of mechanics as a rule that allows him to refer phenomena in time to a conceptual structure that is independent of time. However, it is quite a long way that takes him there. Here is an extremely abbreviated synopsis of Hertz's path to this fundamental law.

The *Principles of Mechanics* divides into 2 books. The first book is entitled "Geometry and Kinematics of Material Systems":

> The subject-matter of the first book is completely independent of experience. All the assertions made are *a priori*-judgments in Kant's sense. They are based upon the laws of the internal intuition of, and upon the logical forms followed by, the person who makes the assertions; with his external experience they have no other connection than these intuitions and forms may have. (PM §1)[2]

In this first book Hertz provides formal definitions of time, space, and mass, and introduces material systems, the notions of possible and impossible displacements, straightest path, motion, velocity, momentum, acceleration, etc. The second book deals with the "Mechanics of Material Systems":

> In this second book we shall understand times, spaces, and masses to be signs for objects of external experience; signs whose properties, however, are consistent with the properties that we have previously assigned to these quantities either by definition or as being forms of our internal intuition. Our statements concerning the relations between times, spaces, and masses must therefore satisfy henceforth not only the demands of thought, but must also be in accordance with possible, and, in particular, future experiences. (PM §296)[3]

[2] In the remainder of this paper, Hertz's *Principles of Mechanics* (Hertz 1884 and 1956) will be cited PM followed by the paragraph number (§) that is identical for the English and German editions. In the case of preface and introduction, page numbers will be given for the English/German editions (in this order). Translations have been silently corrected where necessary.

[3] Hertz's division of books echoes Kant's division between the *Prolegomena* (or the *Critique of Pure Reason*) and the *Metaphysical Foundations of Natural Science*. According to David Hyder, however, it mirrors the division among the parts of the *Metaphysical Foundations* (Hyder 2003).

After introducing the concepts of time, space, and mass as determined by chronometer, scale, and balance respectively, Hertz states his single law of motion or rest which is to take the place and improve upon Newton's three laws:

> 308. We consider the problem of mechanics to be to deduce from the properties of a material system which are independent of time those phenomena which take place in time and the properties which depend on time. For the solution of this problem we lay down the following, and only the following, fundamental law, inferred from experience.
> 309. **Fundamental Law.** Every free system persists in its state of rest or of uniform motion in a straightest path.
> *Systema omne liberum perseverare in statu suo quiescendi vel movendi uniformiter in directissimam.* (PM §§308–309, compare §§119f. and 307).

While it is fairly easy to see that the fundamental law was developed within a geometrized system for the representation of the principles of mechanics, it is not so clear as yet how this law can serve as a plan or rule for the representation of mechanical motions. It does so by orienting mechanics toward the natural motions within the totality of conceivable motions. In other words, it is a method of picking out the natural motions within a wider conceptual framework. With the general considerations of Book 1 thus began a gradual process of determining what is really the case. Book 1 began with conceivable or thinkable motion and used geometrical considerations to distinguish among the conceivable motions those that are physically or mechanically possible and impossible (PM §§11, 109–111). The kinematics of Book 1 continued this process by introducing a further constraint. Among the possible motions of a system, only those are lawfully possible which are independent of time, that is, possible at all times (PM §§119–120, 226–229). Among these lawful motions, finally, those consistent with the fundamental law are picked out as the natural motions (PM §312). The introduction of the law thus concludes a process of gradually zeroing in on natural motion, and the remainder of Book 2 is meant to show that "natural motion" is not only consistent with the fundamental law, but that it includes all and only actual motion (see PM, 39f./46f.).

The purpose of the law is therefore to provide something like a selection parameter that restricts science to the representation of all and only actually obtaining states of affairs, a representation that ascribes neither too much nor too little to the world. As a completely general fact, the law orients, economizes, or systematizes the description of the world. Accordingly, Hertz rejects the notion of a "complete explanation":

> The complete explanation of the phenomena of the material world would [. . .] comprise: (1) their mechanical or physical explanation; (2) an explanation of the fundamental law; (3) the explanation of those properties of the material world which are independent of time. The second and third of these explanations we, however, regard as beyond the domain of physics. (PM §314)

Rather than explain why something is this case or justify the choice of a system of representation, Hertz offers only a mechanical or physical explanation that refers a given event in time to a system of representation which can pick it out as an actual motion and which can then calculate the further trajectory of this motion. It is thus the ability of the fundamental law to organize a complete description of all mechanical phenomena that makes it a law of nature. It has no further and no "deeper" explanatory role to play other than to coordinate features of the system of representation with features of the world as the system that is to be represented. This geometrization of mechanics replaces contingency by necessity; it thereby erases the temporal character of motion and, in a sense, time altogether.

As we have seen Hertz states that the problem of mechanics is to refer the phenomena which take place in time to time-independent properties of material systems (PM §§307–308). He does so by treating the passage of time not as a measurable reality against which other processes can be compared. Instead, the successive states of one material system (a chronometer) are co-ordinated with those of another (a system of points). This arbitrary co-ordination serves as a law of projection or transformation "by means of which we translate external experience, that is, concrete sensations and perceptions, into the symbolic language of our inner picture" (§302). While this rule of projection does not keep us from imagining "time" as a continuous quantity that is not itself dependent on any mechanical process, it renders senseless for the purposes of mechanics the question whether the chronometer provides a true or absolute measure of time: It is sufficient for the rule "to determine such measures as enable us to express without ambiguity the results of past and future experiences" (§304). As "time" becomes through coordination with a material devive part of our picture of presently given external objects, that picture becomes answerable to experience: We can predict that the system of points will change from state A to state B as the chronometer moves from its state A to state B. Not every imaginable system of points will yield to such predictions. But armed with Hertz's mechanics, experimental physics can distinguish within "the infinite world of phenomena" those "finite groups of masses which can exist independently as free systems" (§307) and to which the fundamental law applies straightforwardly.[4]

[4] Compare Hertz's definitions of "time" in Books 1 and 2. Book 1 provides a Kantian definition of time as *a priori* constitutive of experience: "The time of the first book is the time of our internal intuition. It is therefore a quantity such that the variations of the other quantities under consideration may be regarded as dependent upon its variation; whereas in itself it is always an independent variable" (PM §2). Book 2 specifies in which way temporal processes can become subject to scientific experience: "Time, space, and mass in themselves are in no sense capable of being made the subjects of our experience, but only definite times, space-quantities, and masses. Any definite time, space-quantity, or mass may from the result of a definite experience. We make, that is to say, these conceptions signs for objects of external experience in that we settle by what sensible perceptions we intend to determine definite times, space-quantities, or masses. The relations which we state as existing between times, spaces, and masses, must then in the future be looked upon as relations between these sensible perceptions. Rule 1. We determine the duration of time by means of a chronometer, from the number of beats of its pendulum. The unit of duration is set by arbitrary convention. To specify any given instant, we use the time that has elapsed between it and a certain instant determined by another arbitrary convention. This rule contains nothing empirical which can prevent us from considering time as an always independent and never dependent quantity which varies continuously from one value to another. [...] The three foregoing rules are not new definitions of the quantities time, space, and mass which have been completely defined previously. They present rather the laws of transformation

14.3 Limitations of the Fundamental Law

Hertz's introduction of the fundamental law is followed by considerations of its validity and scope, of its limitations and applicability. As for validity and scope, Hertz provides the following classification: First, there is a class of systems of bodies, notably the free systems[5], to which the law applies straightforwardly. "With regard to this class," Hertz writes, the fundamental law "represents a bare experiential fact" (PM §316). Conversely, since the fundamental law can solve any given problem regarding motion in these systems, such a problem would simply *be* a mechanical one (PM §324). To a second class of systems of bodies, the fundamental law can only be applied if appropriate assumptions are made. For certain unfree systems these assumptions involve hidden masses and motions. As regards this class, the fundamental law has the character of a permissible and probable hypothesis (PM §317).[6] Conversely, problems arising for this class can be *regarded as* mechanical problems (PM §325). The third and last class comprises those systems of bodies whose motions cannot be represented as consequences of the fundamental law. "Among these," Hertz writes, "are included all systems which contain organic or living beings" (PM §318). For this class, the fundamental law has the character of a permissible hypothesis. Conversely, if the permissible hypothesis proves insufficient to answer a question regarding the motion of such a system, "there must be a contradiction between the presuppositions of the question and the fundamental law," and "the question posed then cannot be considered a mechanical problem at all" (PM §318). In other words, if the fundamental law can solve a problem, that problem is or

by means of which we translate external experience, that is, concrete sensations and perceptions, into the symbolic language of the pictures of them which we form, and by which conversely the necessary consequents of this picture are again referred to the domain of sensible perceptions. Thus, only through these three rules can the signs time, space, and mass become parts of our pictures of external objects. Also, only by these three rules are they subjected to further demands than are necessitated by our thought. [...] There is, nevertheless, some apparent warrant for the question whether our three rules furnish true or absolute measures of time, space, and mass, and this question must in all probability be answered in the negative, inasmuch as our rules are obviously in part fortuitous and arbitrary. In truth, however, that question needs no discussion here, not affecting the correctness of our statements, even if we attached to the question a definite meaning and answered it in the negative. It is sufficient that our rules determine such measures as enable us to express without ambiguity the results of past and future experiences. Should we agree to use other measures, then the form of our statements would suffer corresponding changes, but in such a manner that the experiences, both past and future, expressed thereby, would remain the same" (PM §§297–298, 302, 304).

[5] "A material system which is subject to no other than internal and lawful connections is called a free system." (PM §122)

[6] Hertz appeals to hidden cyclical motions (PM §§594ff.) when he treats of systems "in which actions-at-a-distance, the forces of heat and other causes of motion, not always fully understood, are in operation. When we bring to rest the tangible bodies of such systems, they do not remain in this state, but on being set free enter into a state of motion again. Thus, apparently, they do not obey the law." Also, systems in which impulses occur do not satisfy the condition of continuity. In this case, the law can be applied on the "exceedingly probable hypothesis that all discontinuities are only apparent and vanish when we succeed in taking into consideration sufficiently small space- and time-quantities" (PM §317).

can be regarded as mechanical, and if a problem is mechanical, the law can solve it. Hertz therefore adds to his description of the three cases the following observation:

> [t]he fact that we may employ the fundamental law in the manner we do, is not to be regarded as a new experience in addition to the law, but is, as we have seen, a necessary consequence of the law itself. (PM §330)

Hertz's fundamental law is formulated in such a way that it picks out what it represents. That the world is describable by this law is entirely trivial – and this, of course, is a virtue of the law when one seeks a complete and true scientific representation that does not aspire to add content or depth but merely to describe. Though the fundamental law does not add anything to the synoptic description of the world and in that sense says nothing, it *does* say something about the world that there are some systems to which it applies immediately, others to which it applies if certain additional assumptions are introduced, and perhaps yet others which must be conceptualized as nonmechanical since to them the law may not apply at all.

If there are nonmechanical systems to which the law does not apply, these would be systems that include living organisms. According to the fundamtental law, mechanical systems are characterized by a simple continuance of motion or rest. In contrast, organisms appear to spontaneously generate motion and the fundamental law cannot describe this. Hertz must therefore ask about systems that include within them a living been that might spontaneously introduce a push or a pull upon some obejct. In that case, the question of the interaction between mechanical and nonmechanical systems might arise, a question addressed by Hertz in two remarks on the "Limitation of the Fundamental Law":

> 320. In a system of bodies which obeys the fundamental law there is neither any new motion nor any cause of new motion, but only the continuance of the previous motion in a given simple manner. One can scarcely help denoting such a material system as an inanimate or lifeless one. If we were to extend the law to the whole of nature [...] — "The whole of nature pursues with uniform velocity a straightest path," — we should offend against a feeling which is sound and natural. It is therefore more cautious to limit the probable validity of the law to inanimate systems. (PM §320)

Even when the probable validity of the law is limited to inanimate systems, however, this precludes in no way that one can still consider animate systems as conformable to the law (see PM §§317–318). Only "if it could be proved that living beings contradict the hypothesis [...] they would separate themselves from mechanics":

> In that case, but only in that case, our mechanics would require supplementing with reference to those unfree systems which, although themselves lifeless, are nevertheless parts of such free systems as contain living beings. (PM §321)

In other words, if living beings contradict the fundamental law, Hertz's mechanics would need to be amended to accommodate, for example, a human being who pushes or throws an object. Hertz suggests that this amendment could be formed

> from the experience that animate systems never produce any different results on inanimate ones than those which can also be produced by an inanimate system. Thus it is possible to substitute for any animate system an inanimate one; this may represent the former in any particular problem under consideration, and we may demand its specification in order to turn the given problem into a purely mechanical one. (PM §321)

Thus, if it should turn out that human or other animate beings are not mechanical systems and contradict the fundamental law, they can be represented by robots when they have an effect on an inanimate system, for example, by providing some kind of push: While their inner workings would be blackboxed as unmechanical, their interventions in inanimate material systems could still be treated as purely mechanical. Again Hertz adds an observation:

> In the usual presentation of mechanics such a qualification is considered superfluous and it is assumed with certainty that the fundamental laws extend equally to animate and inanimate nature. And this is indeed permissible in that usual representation, since one leaves the greatest degree of freedom to the forms of the forces which there enter into the fundamental laws, and reserves oneself an opportunity of explaining later and outside of mechanics, whether the forces of animate and inanimate nature are different, and what properties may distinguish the one from the other. Our presentation of the subject demands greater caution, since a considerable number of experiences which initially relate only to inanimate nature are already included in the fundamental law itself, and the possibility of a later demarcation is much more limited. (PM §322)

Hertz here underscores an important difference between his own and, for example, Newtonian mechanics. In the Newtonian picture, "force" appears right at the beginning as an undefined primitive term. Whatever impinges upon or alters a straight trajectory of continuing motion is a force, and this force may turn out to be gravitational attraction or a volitional push. In Hertz's mechanics, the term "force" is a derived concept that is introduced only late in the game. It serves merely as a short-hand expression once one considers the combined motion of coupled material systems: If the coordinates of two systems are determined in such a manner that one or more coordinates of one system remain equal to one or more coordinates of the other system (PM §450), if one then considers how each system persists in its state of rest or uniform motion (that is, conforms to the fundamental law), one can conceptualize as "force" the influence that one of the systems appears to independently exert upon the motion of the other (PM §455).[7] On this conceptualization of force, "one can scarcely help denoting" that each of the coupled material systems is "an inanimate or lifeless one" since each represents nothing but "the continuance of the previous motion in a given simple manner" (PM §320). Moreover,

[7] This paraphrase renders Hertz's definition more accurately than does the extant English translation: "Unter einer Kraft verstehen wir den selbständig vorgestellten Einfluß, welchen das eine von zwei gekoppelten Systemen zufolge des Grundgesetzes auf die Bewegung des anderen ausübt" (PM §455).

since force is restricted to contact-force, force and counter-force are from the start conceived as arbitrarily interchangeable (PM §456), preventing Hertz from assigning "cause" to one and "effect" to the other. His is thus a mechanics not just without force but also without causality and the fundamental law of his mechanics cannot be reduced to or derived from the causal law. But all of this puts his system at odds with "a feeling which is sound and natural" about human action that can spontaneously cause physical motions to occur.[8]

When Hertz acknowledges that the usual presentation of mechanics needs no qualifications and applies to animate and inanimate nature alike, while his presentation requires greater caution to accommodate animate systems, this may sound as if he was admitting a defect of his mechanics. However, the apparent defect turns out to be a virtue and the virtue is precisely that Hertz does not need to venture outside of mechanics to consider the nature of force in animate systems.

14.4 The Virtue of Limitations

According to Allan Janik, Hertz is committed to a principle of immanence: "[I]f philosophical problems arise *in* physics, then they must be handled in physics itself rather than in some theory *about* physics" (Janik 1994/95, 23). Such a concern with immanence does indeed motivate Hertz's *Principles of Mechanics* and indicates what the virtue of his mechanics is – especially since it allows him to delimit the domain of mechanics (and thus of science), leaving the "problem of life" outside of this domain.

It is well known that Hertz used the three criteria of (logical) permissibility, (empirical) correctness, and appropriateness when assessing the virtues of his own against two standard presentations of mechanics. About permissibility, all that needs to be said here is that where Hertz saw possible contradictions in the usual Newtonian presentation, he argues that his mechanics is free of contradiction.[9] As for correctness, the three pictures of mechanics can be taken to be empirically equivalent — at least as concerns the then-present state of knowledge.[10] This leaves finally the question of how appropriate a presentation of mechanics is, and this requires an elaboration of what Hertz means by "appropriateness".

All presentations of mechanics are certain kinds of pictures which have an internal dynamic structure through which informational input is processed into informational output. The relations between these inputs and outputs Hertz calls "essential relations." Mechanics has to get these essential relations right: The relationship between observational antecedents and consequents needs to agree with the represented or pictured an-

[8] Also, Hertzian forces always satisfy conservation of energy while he takes spontaneous action and freedom of the will to be non-conservative (PM pp. 10f./11f., 39/56f. and §§662ff.; compare Manno 1900, 42–46).

[9] Murat Baç maintains that permissibility serves as Hertz's decisive criterion (Baç 2000, 44 and 52). Others disagree whether Hertz really took issue with the permissibility of the Newtonian picture (Nordmann 1998a, 170 and Hentschel 1998, 219).

[10] At the very end of his Introduction to the *Principles* Hertz discusses what kind of evidence might decide between the various systems of mechanics (Nordmann 1998a, 163f.).

tecedents and consequents.[11] However, the internal structure of the picture also requires for the transformation of input into output "empty relations." These empty relations do not correspond to anything observable in nature but are necessary to generate accurate predictions (PM 2/2f.). Now, one theory is more appropriate than another i) if it contains fewer empty relations, that is, if it is simpler, and ii) if it sharply distinguishes the essential from the empty relations, that is, if it is more distinct.[12]

As for simplicity of use, Hertz readily admits that his conception of mechanics is rather complicated in practical contexts.[13] And as for conceptual simplicity, many of his contemporary critics as well as today's readers have wondered why one should consider simpler a mechanics that reduces the number of primitive terms by one, namely force, at the expense of appealing to hypothetical entities instead, namely hidden motions and masses (Mulligan 1998). According to Hertz, however, this trade-off reduces the number of empty relations: It pares down the conceptual apparatus of mechanics. "If we try to understand the motions of bodies around us," he explains,

> [w]e soon become aware that the totality of what we can see and grasp does not form a world conformable to law, in which the same conditions always have the same consequences. [...] If we wish to obtain a picture of the world which shall be well-rounded, self-contained, and conformable to law, we have to suppose, behind the things which we see, other, invisible things — to imagine confederates concealed beyond the limits of our senses.

A mechanical picture of the world thus requires the introduction of "empty relations," that is, relations between secret players who act behind the scenes and whose actions lie beyond the limits of sense and are therefore inaccessible in principle to experience. Hertz continues:

> These deeper-lying influences we acknowledged in the [traditional presentations of mechanics]; we imagined them to be entities of a special and peculiar

[11] Compare Hertz's famous statement: "We form for ourselves pictures or symbols of external objects; and we make them in such a way that the necessary consequents of the pictures in thought are always the pictures of the necessary consequents in nature of the things pictured" (PM 1/1). Upon closer scrutiny, this statement uses the term 'picture' ambiguously (and that ambiguity is dissolved only later when Hertz identifies "pictures" with "dynamic models"). Also, it appears to suggest that there is a necessity in nature that is somehow independent of or prior to its pictorial representation. Only the consideration of Hertz's further elaborations will clear this up.

[12] In his lectures regarding the constitution of matter, Hertz ran up against the problem that all conceptions of matter run together *a priori* and empirical elements – that is, that there is no distinct conception of matter (Hertz 1999).

[13] "The appropriateness of which we have spoken has no reference to practical applications or the needs of humankind. In respect of these latter it is scarcely possible that the usual representation of mechanics, which has been devised expressly for them, can ever be replaced by a more appropriate system. Our representation of mechanics bears towards the customary one somewhat the same relation that the systematic grammar of language bears to a grammar devised for the purpose of enabling learners to converse as quickly as possible about the necessities of daily life. One knows how different the requirements for these two purposes are, and how widely their arrangement must differ if each is to be properly adapted to its purpose" (PM 40/47f.). — This passage is central to Janik's contention that Hertz rehearses a hermeneutics of representation (Janik 1994/95).

> kind, and so, in order to reproduce them in our picture, we created the concepts of force and energy. But another way lies open to us. We may admit that there is a hidden something at work, and yet deny that this something belongs to a special category. We are free to assume that this hidden something is nothing else than motion and mass all over again, — motion and mass which differ from the visible ones not in themselves but in relation to us and to our usual means of perception. (PM 25/30)[14]

Once hidden entities have to be assumed, the simpler of two theories is the one that contains fewer empty relations and that does not ascribe mysterious properties to these hidden entities. In any event, the hidden masses and motions in Hertz's mechanics are hypothetical not in the sense that he posits them as objects of some conceivable future experience with vastly improved observational technology. Just like force or energy, they are hypothetical in that they are internal structural elements of the picture, of the net, not meant to pick out anything in the world.[15]

If simplicity concerns the number and nature of empty and ineluctably hypothetical relations, distinctness refers to the sharp separation of empty and essential relations, and thus to the precise determination of essential relations. Hertz's own presentation of mechanics is said to be distinct because it succeeds at delimiting the set of natural motions, admitting neither too much nor too little, while Newton's presentation admits motions that do not occur in nature and it thus fails to clearly specify those that do (PM 10/11f. and 39/46f.).[16] The criterion of distinctness thus expresses Hertz's interest in immanence as it was identified by Janik. Aside from being "well-rounded" and "conformable to law," the physical picture of the world is to be "self-contained" (PM 25/29f.): Science knows from experience what the natural motions are and will construct any number of

[14] Poincaré paraphrases this passage, adding: "N'est-ce pas plus simple et plus naturel?" By way of commentary, he then introduces a further consideration: If we follow Hertz and admit only non-mysterious entities in physics, we are perhaps more likely to delude ourselves that we can already explain everything: "On pourrait discuter sur ce point et soutenir que les entités des anciennes théories doivent être retenues précisément à cause de leur nature mystérieuse. Respecter ce mystère, c'est un aveu d'ignorance; et puisque notre ignorance est certaine, ne vaut-il pas mieux l'avouer que la dissimuler?" (Poincaré 1897, 247). The previous discussion has shown, however, that Hertzian physics limits itself in such a way that it cannot give rise to this delusion.

[15] Here I am disagreeing with John Preston's contribution to this volume and his suggestion that Hertz is a realist regarding hidden masses and motions, material and mass points. After defining "mass" in PM §300, Hertz adds in §301: "We also admit the idea that in addition to the bodies which we can handle there are also other bodies which we can neither handle, move, nor place on the balance and to which [the rule for the definition of mass] has no application. The mass of such bodies can only be determined by hypothesis. As concerns such masses that are to be assumed hypothetically, it is in our power to attribute nothing to them through the hypothesis which would contradict the properties of the conceptually defined mass." Especially the last sentence suggests that Hertz here prepares the ground for the discussion later on of hidden masses (PM §§595ff): These hidden masses are inaccessible to any physical experience whatsoever. No matter how much the technologies of weight-determination improve, the masses defined in $301 elude their grasp (PM §598).

[16] According to Hertz, the Newtonian presentation (in its original form) permits forces that violate the conservation of energy. Also, it is not limited just to such forces that are the sum of the actions of elementary forces with specific properties (PM 10/11f.).

systems of mechanics accordingly. Only a self-contained system assures, however that one does not have to step outside it anymore and consult experience to determine which of the permitted motions actually occur. Hertz therefore demands, that the limits of mechanics and its domain of application are drawn from within mechanics itself. The virtue of distinctness thus extends from the delimitation of the natural motions to the delimitation of the questions and problems of mechanics.

All of this allows us to see the various ways in which Hertz's elimination of "force" as a basic concept renders mechanics simpler and more distinct: The number of empty relations is reduced, and mechanics becomes more self-contained and no longer gives rise to vexing questions which cannot be answered by mechanics:

> [W]e have heaped upon the signs "force" and "electricity" more relations than can be completely reconciled amongst themselves; we have an obscure feeling of this, demand enlightenment, and express our confused wish by posing the confused question as to the nature of force and electricity. But this question is obviously mistaken in regard to the answer it expects. It cannot be satisfied through knowledge of novel and yet more relations and connections, but by removal of the contradictions among the extant ones; thus perhaps by reduction of the number of extant relations. When these painful contradictions are removed, the question as to the nature [of "force" and "electricity"] will not have been answered; but our mind, no longer vexed, will cease to ask what for it is an illegitimate question. (PM 7f./9)

The question concerning the nature of force is not the only philosophical problem solved or dissolved by Hertz as he articulates a distinct conception of mechanics, one that specifies its own limits. Echoing the previously quoted remarks on the limitation of the fundamental law, Hertz notes in his Introduction that "[I]t is surely a justified caution when in the [body of the] text we expressly limit the domain of our mechanics to inanimate nature, leaving it an entirely open question how far its laws extend beyond it" (PM 38/45). He continues here by elaborating why he considers this limitation of mechanics not a defect, but a virtue:

> [T]he same feeling which impels us to exclude from the mechanics of the lifeless world as foreign every suggestion of an intention, of a sensation, of pleasure and pain, – this same feeling makes us hesitate to deprive our picture of the animated world of these richer and more colorful conceptions. At least to a cursory estimation, our fundamental law – perhaps sufficient to represent the motion of dead matter – appears too simple and too limited to reproduce the manifold character of even the lowest process of life. That this is so seems to me not a disadvantage but rather an advantage of our law. Precisely because it affords us to synoptically view the whole of mechanics, it also shows us the limits of this whole. Precisely because it gives us only a fact without attributing to it the semblance of necessity, it lets us recognize that everything could also be different. (PM 38/45)

To see the limits of mechanics, to see the fundamental law as a completely general, yet merely descriptive proposition that applies only contingently to the world, avoids conceptual confusion.[17] Again, Hertz makes the contrast to the customary presentations of mechanics, in which

> the complete vagueness of the forces introduced leaves considerable degrees of freedom. There is a tacit proviso perhaps to determine later on a contrast between the forces of animate and inanimate nature. In our presentation, the outlines of the picture are from the start so sharply delineated, that deeply pervasive divisions can hardly be introduced retroactively. (PM 38f./45f.)

Hertz thus insists twice – in his Introduction and the body of the work – on the clear demarcation in his mechanics between the animate and the inanimate. He divorces the entire domain of physics from the problem of life. He thereby divorces from physics also the then most popular philosophical speculation about the nature and limits of science, namely the discussion prompted by Emil DuBois Reymond's *Die sieben Welträtsel* of 1882.[18] That discussion lasted well into the twentieth century, and the *Lebensproblem* or problem of life was among DuBois Reymond's seven riddles or scientifically intractable puzzles. Its discussion involved not only mechanism, vitalism, and energeticism, but also anthropomorphized conceptions of physical force on the one hand (PM 12/15), and on the other hand the promise of science to identify the life force which distinguishes animate from inanimate nature.[19] By divorcing physics from this host of issues, Hertz rejects such physical or philosophical temptations.[20]

[17] This formulation only appears to contradict the remark above according to which the fundamental law offers a complete account that is trivially true: The law would be true of all mechanical motion even if no such motions were to be found. But it is a contingent experiential fact that all natural motions of inanimate systems can be treated as mechanical motions by the fundamental law. (This tension is characteristic of all conceptions of law in the Kantian tradition.)

[18] Hertz knew DuBois Reymond and his work (Fölsing 1997, 82, 111).

[19] Compare PM 6/7 where Hertz suggests an altogether unfavorable parallelism between the "*lebendige Kraft*" or *vis viva* and centrifugal force, which – according to Hertz – "takes the effect of inertia into consideration twice, namely first as mass, secondly as force." He continues: "In our [Newtonian] laws of motion force was the cause of motion prior to the motion. May we now suddenly speak, without confusing our concepts, of forces which only arise through motion and which are a consequence of motion? May we create the impression as if our laws had already said something about this new kind of forces, as if by calling something 'force' we already endow it with the properties of force?" If, as in the case of *vis viva*, this centrifugal force is an "unreal" force, our calling it force cannot be justified, only "excused" by appeal to "historical tradition." And in both cases, what gives rise to the confusion is the temptation to anthropomorphize, that is, to animate the notion of "force," for example, by unduly projecting the experience of whirling a stone at the end of a rope into the orbits of satellites around central bodies (PM 12/15). – According to Simon Saunders (and perhaps Ernst Mach), Hertz's conceptual critique of centrifugal force constitutes an "embarrassingly bad argument" (Saunders 1998, 152, compare PM 8f./9f.).

[20] A more detailed examination would relate Hertz's distaste of DuBois Reymond's *Welträtsel* to his distinterest in sense physiology, comparative psychology, and Darwinism. This investigation would also consider his relation to Helmholtz and Mach, his dismissive attitude toward psycho-physics, and (especially relevant for the *Principles of Mechanics*) his criticism of Helm's and Ostwald's energeticism (for all of this see Fölsing 1997).

14.5 The Mechanics of Representation

On first sight, Hertz's argument is directed only against the vitalisms and mechanicisms of his day, and he therefore concerns himself only with the possibility, in principle, of non-mechanical systems, with the intellectual illegitimacy of a so-called life force or otherwise anthropomorphized conceptions of mechanical force, and with the inability of mechanics to account for even the lowest processes of life. As we have seen already, his argument against certain tendencies of his day was motivated by a larger philosophical program. Hertz wants to sharply distinguish essential from empty relations, that is, to discover in our pictures of the world what in them refers to "an inner intellectual world of conceptions and ideas" with its "colorful" attributes of "intention, sensation, pleasure and pain," and what in these pictures refers to "the cold and alien world of actual things" and "the motion of dead matter" (PM 2f./3, 38/45f. and Hertz 1896, 335). Working from within the descriptive science of mechanics, this program ends up taking Hertz beyond mechanics to an account of what is knowable by any science.

When Hertz identifies the limits of mechanics with the limits of physics and the limits of science, he does this without generalizing from mechanics to other sciences, and without invoking a metaphysical reductionism according to which everything boils down to physics, in general, and mechanics, in particular. Instead, mechanics serves Hertz as the paradigm for any and all descriptions of the world: The ontological presuppositions and epistemological limits of mechanics coincide with the ontological presuppositions and epistemological limits of all descriptive representations of the world. [21]

As we have seen, the fundamental law offers a synoptic description of all natural motion: It provides a rule by which features of a physical state can be coordinated with features of a model or system of representation. The descriptive power as well as the limits of mechanics are thus associated with a general feature of all scientific representation, namely the making of a model that corresponds to a natural system. Hertz shows that this kind of modeling doesn't just happen in mechanics as it does in all the other sciences. Instead, mechanics elucidates the general relation of constructed model and the state of nature that is to be represented and that becomes presentable through the representation. The very idea that our mind can create dynamical models which describe and predict natural motions by referring processes in time to time-independent structures, invokes a mechanics of representation which simultaneously underwrites the principles of mechanics and is elucidated, if not justified by these principles.

Hertz establishes all this in a section of the *Principles of Mechanics* that deals with "Dynamical Models". Their definition includes the requirement that two material systems are models of one another only if they have the same number of coordinates (PM § 418). If the relationship between a representation and a represented state of nature is defined

[21] Jesper Lützen attributes to Hertz the view that all of nature is "one connected mechanical system" (Lützen 2005, 5 and see also 30, 38–39, 191). Hertz can surely be called a mechanicist in that he takes nature to be a mechanical system to the extent that it is scientifically intellible at all (and *vice versa*). However, Hertz did not hold the metaphysical position that all of nature could or should be reduced to mechanics.

by their sharing the same number of distinctive characteristics, their relationship is symmetrical. In a first corollary to his definition, Hertz therefore notes:

> If one system is a model of a second, then, conversely, the second is also a model of the first. If two systems are models of a third system, then each of these systems is also a model of the other. The model of the model of a system is also a model of the original system. (PM § 419)

The second corollary furthers the impression that one cannot get out of a system of representation once it is established. The definition of the dynamical model does not allow for any comparative judgment about the two systems, that is, the model exists just as soon and only as long as the required correspondence of coordinates happens to obtain. Hertz writes:

> The property of a system of being a model of another does not depend on the choice of co-ordinates of the one or the other system, even though this property becomes immediately evident only upon a certain choice of co-ordinates. (PM § 420)

In a final corollary Hertz notes that "a system is not completely determined by the fact that it is a model of a given system," since, in order to be dynamically similar, the two systems need not consist of the same kind of stuff (PM § 421). Indeed, a representational system consists of symbols whereas the represented system consists of material points. In order to be models of one another, the two systems need to be dynamically similar only in the one respect of the chosen coordinates and can be different in all others. Their similarity-relation is the essential relation, it either obtains or it does not obtain, and this can be determined unambiguously. Their differences, in contrast, remains opaque and unknowable:

> [I]t is generally impossible to carry our knowledge of the connections of natural systems further than is involved in specifying models of the actual systems. We can then, in fact, have no knowledge as to whether the systems which we consider in mechanics agree in any other respect with the actual systems of nature which we mean to be considering, than in this alone, — that the one set of systems are models of the others. (PM § 427)

Since the various mechanical pictures of the world also contain empty relations, namely assumptions about forces or hidden masses and motions, one cannot know whether agreement with nature extends to these assumptions. Accordingly, Hertz derives from his mechanics a general epistemological conclusion regarding the limits of knowledge:

> The relation of a dynamical model to the system of which it is regarded as the model, is the same as the relation of the pictures our mind forms of things to these things. [. . .] The agreement between mind and nature can therefore be compared to the agreement between two systems which are models of one another, and we can even account for this agreement by assuming that the

> mind is capable of making actual dynamical models of things and of working with them. (PM § 428)[22]

Again we are discovering that Hertz's mechanics is distinct in that it is self-limiting. This time, it does not limit itself to inanimate nature but limits any knowledge of nature to knowledge of the dynamic similarity between the constructed representation and the represented system. By sharply distinguishing between essential and empty relations Hertz's mechanics determines not only the scope of a particular theory but a general limit on what we can know. It does not just happen to be the case that mechanics cannot describe even the lowest processes of life, but the very project of gaining descriptive knowledge of nature by creating in our minds dynamical models is constrained by mechanics. After all, these dynamical models can serve the most important and elementary task of science, namely "the anticipation of future events" (PM 1/1) only if they satisfy Hertz's *Lehrsatz* or theorem in the section on Dynamical Models:

> If two systems, each of which is a model of the other, have corresponding conditions at a definite time, then they have corresponding conditions at all times. For by the equations of condition of a system, the expression for the magnitude of displacement and the initial values of the coordinates and their changes, the course of the coordinates is determined for all times [...] (PM § 424).

In other words, in a successfully predictive science the representational model and the represented system will both exhibit "only the continuance of the previous motion in a given simple manner." And as before, one can therefore "scarcely help denoting" that such a simple continuance of motion is a feature only of "inanimate or lifeless" systems (PM § 320).

All this can be formulated more plainly: In order to generate predictions, scientists extrapolate or in some other manner think through their models to arrive in a more or less mechanical fashion at a determinate conclusion. In the meantime, the physical system of nature develops along the determinate pathways of its dynamics. Predictive knowledge is possible only when these two mechanical, lifeless motions unfold in parallel – any kind of spontaneity or sign of life would disrupt the parallelism and take us out of the domain of mechanics, of inanimate nature, and of the agreement of mind and nature by means of representation. Thus, the scientific pursuit of representing the world is basically a lifeless activity – it will only succeed and predictive scientific knowledge will only be possible if these descriptive models are properly geometrized

[22] To be sure, in order to meet the requirement of "dynamical model," the hypothetical motions of hidden masses must be attributed to the systems of nature themselves. Along with measurable co-ordinates these systems of nature must therefore be said to have co-ordinates that are unmeasurable in principle (see note 15 above). This creates a conceptual difficulty for Hertz since his mechanics provides a model of nature only in reference to the measurable co-ordinates. This difficulty can be avoided, however, if one allowed for internal empty relations to be black-boxed: Being empty, they do not specify "co-ordinates" at all and do not enter into the modeling relationship.

and the spontaneity of linguistic expressiveness is suppressed. If the aim of science is to create lifeless representations, such representations are also incapable of representing living processes that express spontaneous actions. This is the price to pay if one seeks genuine agreement between mind and nature as two dynamic systems: Their agreement presupposes that each continues in a mechanical manner. If this condition is met, all that needs to be done and all that can be done by science is to properly calibrate these systems such that they continue each in their simple but now coordinated manner.

While this specific epistemological lesson emerges from Hertz's distinct presentation of mechanics, he applies it to all theorizing, including the customary presentations of mechanics. This is why the conclusion of Hertz's book can appear right at its outset. Indeed, his very project of comparing three empirically equivalent pictures of mechanics is underwritten by his epistemological conclusion according to which all three are nothing but dynamical models of the system of nature.

> We form for ourselves pictures [*Scheinbilder*] or symbols of external objects; and we make them in such a way that the necessary consequents of the pictures in thought are always the pictures of the necessary consequents in nature of the things pictured. In order for this requirement to be satisfiable at all, there must be certain agreements between nature and our mind. [...] The pictures of which we are speaking are our conceptions of things; they agree with the things in *one* important respect which consists in satisfying the above-mentioned requirement, but for their purpose it is not necessary that they are in any further agreement with the things. Indeed, we do not know, nor have we any means of discovering whether our conceptions of things agree with them in anything at all but in this *one* fundamental relation alone. (PM 1f./1f.)

The philosopher Heinrich Hertz thus delimits a contingent world of representation where the coordination between representational models and represented states of nature is effected by dynamical models that are constructed according to one plan, namely a fundamental law that is formulated for a geometry of systems of points. In this world, scientific knowledge is possible. Hertz thus accounts for the possibility of a natural science without the metaphysical presupposition of natural or causal necessity. At the same time, he juxtaposes the world of representation in which all problems are assumed to be physical, if not mechanical, and all motion simply continues, to a world of will in which one confronts spontaneity, the problem or problems of life, of intention and sensation, pleasure and pain.

14.6 Coda

This paper set out by claiming that it avoids the detour through other philosophers who were influenced by Hertz, such as Ludwig Wittgenstein. In light of the proximitiy of their views, Wittgenstein scholars might find this claim less than ingenuous and wonder how much of Wittgenstein's philosophy was here read back into Hertz's work. While certain turns of phrase in this reconstruction inevitably owe to my own previous attempts to discuss the relation of Wittgenstein and Hertz (Nordmann 2005), I tried to counteract this suspicion precisely by presenting the coherence of Hertz's philosophical positions in its own terms.

A second, perhaps complementary paper would be required to elaborate the striking closeness between Hertz's philosophical views and those, in particular, of Wittgenstein's *Tractatus Logico-Philosophicus* (Wittgenstein 1922).[23] This would begin with the notion that mechanics provides a single plan by which to construct all true propositions required for the description of the world (TLP 6.343) and thus the idea of a gradual determination of all actual states of affairs from the collection of all possible ones (and with explicit reference to Hertz TLP 6.361 ...) – and the rejection of "explanations" that profess to go beyond such a determination of the actual (TLP 6.371, see also TLP 6.36 on "laws of nature"). Indeed, one can read large portions of the *Tractatus* – including the entire account of completely general propositions, science and explanation, lawfulness and the passage of time – as owing to Hertz, namely TLP 6.34 to 6.3611.[24]

Methodologically, Hertz and Wittgenstein share a commitment to the principle of immanence (Janik 1994/95) and thus to the idea that physics (or logic) must take care of itself (TLP 5.437). Accordingly, Wittgenstein also pursues a program of "delimitation from within" which provides a view of the domain of the sayable as a whole (TLP Preface, 6.45). This enables him to simply and respectfully leave alone all that may or may not exist beyond that limited whole. In particular, the problem or problems of life remain untouched, according to Wittgenstein, once all the questions of science have been answered (TLP 6.52, 6.521). And Hertz's suggestion that illegitimate, yet vexing questions simply go away if a confusing concept like "force" is removed from the basic vocabulary emerges as a paradigm for Wittgenstein's life-long employment of a "therapeutic method" which aims to remove all philosophical problems in quite the same fashion.

This delimitation of the sayable or scientifically answerable proceeds from a consideration of language as a kind of picturing or modelling which works by coordinating sentences and other factors in a "dynamical model" (Wittgenstein explicitly refers to Hertz's use of that term in TLP 4.04). For Wittgenstein, a sentence is a model of a states of affairs and there is no outside vantage point (TLP 4.041) that would allow us to say anything about the relation of sentence and world other than simply to note that – despite all their obvious dissimilarities – one is a model of the other in virtue of the coordination of words in the sentence with things in the states of affairs. These constraints on what we can know make the description and determination of the world

[23] The following paragraphs will cite this as TLP and provide references to specific numbered remarks in that book. – The striking closeness between Wittgenstein and Hertz calls to mind Wittgenstein's most explicit reference to Hertz as an important influence – one that was prefaced by a view of himself as a "reproductive" thinker: "I believe that I have never *invented* a movement of thought, but it was always given to me by someone else. [...] This is how Boltzmann, Hertz, Schopenhauer, Frege, Russell, Kraus, Loos, Weininger, Spengler, Sraffa influenced me" (Wittgenstein 1998, 16).

[24] Arguably and in light of PM §§665–667, this large Hertzian section of the TLP might be starting at 6.33 already.

possible, and also our communication about what is or is not the case as a matter of fact. In other words, though there have been numerous commentaries and analyses written on the relations of Hertz and Wittgenstein,[25] a comprehensive appreciation of Hertz's philosophy in the *Principles of Mechanics* allows us to read in a Hertzian manner numerous propositions of Wittgenstein's *Tractatus* that have not been included in these discussions as of yet (TLP 4.041, 4.11, 5.5262, 6.33, 6.371, 6.342f., 6.361f., 6.432, 6.5, 6.52f.).

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[25] A 1998 bibliography lists 26 items (Nordmann 1998b), an updated version adds another eight, not counting, for instance, several papers forthcoming from a June 2002 workshop on Hertz and Wittgenstein at the College de France. The number continues to grow, with three further additions in the present volume.

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